Notes on Proving Odd, Even, or Neither functions using Algebra

In Algebra 2, we learned these terms and were able to check for symmetry graphically. For example:
A. The function below is EVEN. (The graph is a mirror image of itself over the y-axis.)

B. This function is ODD. (If we turned the graph around the origin – (0,0) – 180 degrees, we’d see the same graph.)

C. While this function is neither.
Look again at Graph A. The algebraic way to consider this function EVEN is to notice that if we randomly chose an x-value and its opposite, the y-values are the same. For example:

\[ f(3) \approx -0.8 \quad \text{and} \quad f(-3) \approx -0.8 \]

Similarly,
\[ f(-2) = 0 \quad \text{and} \quad f(2) = 0. \]

In other words: \( \text{IF} \quad f(x) = f(-x) \quad \text{then, the function is EVEN.} \)

Look again at graph B. The algebraic way to consider this function ODD is to notice that the y-value of a random x-value is the opposite of the y-value for the opposite x-value. For example:

\[ f(0.5) = -1 \quad \text{and} \quad f(-0.5) = 1. \]

In other words: \( \text{IF} \quad f(-x) = -[f(x)] \quad \text{then, the function is ODD.} \)

If neither statement is true, \( f(x) \neq f(-x) \quad \text{AND} \quad f(-x) \neq -[f(x)], \) the function is neither even nor odd.

**PROVING SYMMETRY:**
We would never finish these proofs if we checked every possible x-value. So, we generalize the proofs instead by using variables.

*Some things to remember first:*

1. Given \( f(x), \) when we “plug in” a value or variable for \( x, \) that value or variable should be in parentheses first. Then we can evaluate. :

\[
\begin{align*}
  f(x) &= 4x - 2 \\
  \text{so } f(10) &= 4(10) - 2 = 40 - 2 = 38 \\
  \text{and } f(a) &= 4(a) - 2 = 4a - 2 \\
  \text{and } f(-3) &= 4(-3) - 2 = -12 - 2 = -14 \\
  \text{and } f(x-5) &= 4(x-5) - 2 = 4x - 20 - 2 = 4x - 22
\end{align*}
\]

2. \(-x^2 \neq (-x)^2\) because \(-x^2\) means \(-1 \times x^2\) while \((-x)^2\) means \(-x \times -x = x^2\)

3. \(-x^3 = (-x)^3\) because \(-x^3\) means \(-1 \times x^3\) and \((-x)^3\) means \(-x \times -x \times -x = -x^3\)

Examples of proofs:

Show that \( f(x) = 3x^2 + 7 \) is even.

We must show that \( f(x) = f(-x), \)
\[
\begin{align*}
  3(x)^2 + 7 &= 3(-x)^2 + 7 \quad \text{remember: } (-x)^2 = -x \times -x = x^2 \\
  3x^2 + 7 &= 3x^2 + 7
\end{align*}
\]

Show that \( g(x) = 5x^3 \) is odd.

We must show that \( g(-x) = -[g(x)], \)
\[
\begin{align*}
  5(-x)^3 &= -[5x^3] \\
  5(-x^3) &= -5x^3 \\
  -5x^3 &= -5x^3
\end{align*}
\]
Show that \( h(x) = 2x^4 + x^3 \) is neither.

We must show that

\[
\begin{align*}
  h(x) & \neq h(-x) \\
  2(x)^4 + (x)^3 & \neq 2(-x)^4 + (-x)^3 \\
  2x^4 + x^3 & \neq 2x^4 - x^3
\end{align*}
\]

AND that

\[
\begin{align*}
  h(-x) & \neq -[h(x)] \\
  2x^4 - x^3 & \neq [2x^4 + x^3] \\
  2x^4 - x^3 & \neq -2x^4 - x^3
\end{align*}
\]

Note: We already evaluated \( h(-x) \) above!

**NOTE:** If you use real numbers in your proof, I will write “This is an example not a proof!” on your paper and you will not earn full credit. If you use real numbers rather than variables, you’ve only proven the statement true for that number. You’d have to prove that for ALL real numbers.

For example:
Show that \( f(x) = 3x^2 + 7 \) is even.

We must show that

\[
\begin{align*}
  f(x) & = f(-x). \\
  3(2)^2 + 7 & = 3(-2)^2 + 7 \\
  3(4) + 7 & = 3(4) + 7 \\
  19 & = 19
\end{align*}
\]

This is an example not a proof!